Learning Based Signal Processing for Non-linear Compensation

Ole Winther and Jan Larsen

Informatics and Mathematical Modelling
Technical University of Denmark

www.imm.dtu.dk
owi,jl@imm.dtu.dk

Outline

- General perspectives on using learning based signal processing
- Using learning techniques to improve the physical modeling approach

Learning based signal processing in a nutshell

- From white to black box models
  - Parametric models - physical
  - **Semi-parametric models**
    - parallel models compensate for noise and effects not handled by the physical model
  - Non-parametric models - purely data driven

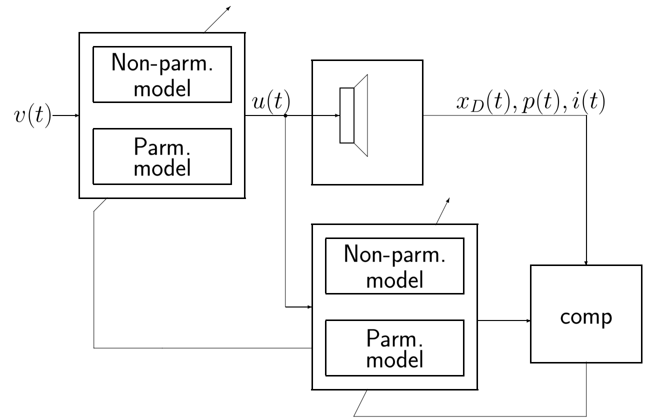
Learning based signal processing in a nutshell

- Bayesian modeling techniques combines learning from data and prior (physical) knowledge
  - ☺ Robustness by model averaging
  - ☺ Careful model structure control and performance
  - ☺ Learning from few data - required if system change relatively fast
  - ☹ Complex if new methods like Variational Bayes is not used
- Adaptive learning of model and controller

Learning based signal processing in a nutshell

- New model structures ensure graceful degradation in nonlinear extrapolation
  - Neural networks - e.g. recursive
  - Kernel methods (Gaussian Processes)
  - Nonlinear state-space models

General compensation system



General compensation system

☺ Parallel non-parametric/parametric structure enable better handling on unknown physical effects and noise
  - maybe a non-par/par state-space model is useful

☺ Forward modeling eliminates problem of non-unique inverse

☺ Possibility of using advanced performance measures - e.g. psycho-acoustical

☺ Bayesian framework ensures optimal combination of data and prior knowledge

☺ New nonlinear models ensure graceful degradation when extrapolation is required

Using learning techniques to improve physical modeling approach

- Loudspeaker master equations
- Main non-linearities
- Fixed feed-forward control
- State space model
- Fitting the non-linearities
- Adaptive feed-forward control

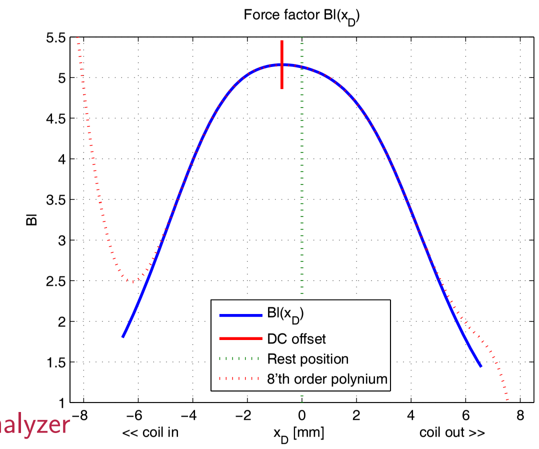
Master equations

- $v_g(t)$: input voltage (= output from amplifier).
- $x_D(t)$: displacement of diaphragm.
- $i_c(t)$: current.

$$\begin{aligned} v_g(t) &= R_E i_c(t) + L_E \frac{di_c(t)}{dt} + Bl \frac{dx_D(t)}{dt} \\ F_c(t) &= M_D \ddot{x}_D(t) + R_D \dot{x}_D(t) + \frac{1}{C_D} x_D(t) \\ F_c(t) &= Bl\, i_c(t) \end{aligned}$$

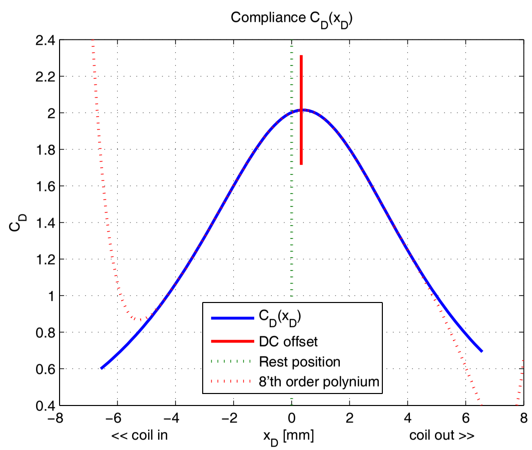
- The non-linearities (NL) are hidden in the x_D and temperature dependence of the 'constants'.

NL I: Magnetic force factor $Bl(x_D)$

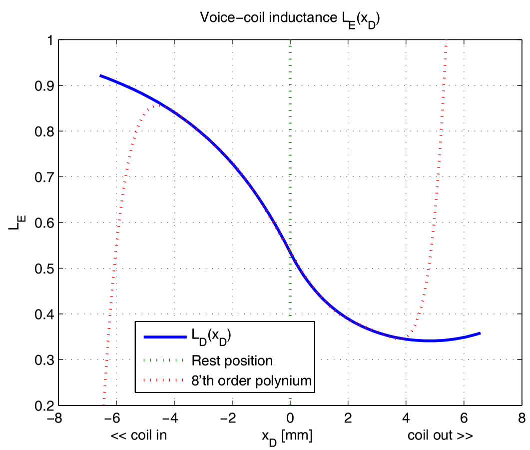


Klippel analyzer

NL II: Compliance $C_D(X_D)$



NL III: Voice coil inductance $L_E(X_D)$



Other variable parameters

- R_E: electrical resistance temperature dependent.
- Magnetic attraction force.
- Others...

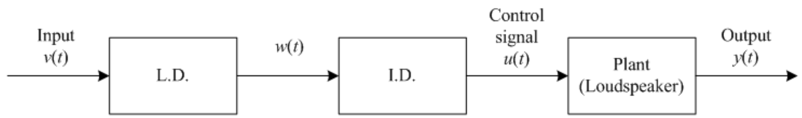
Control of transducer

Compensate for non-linearities to get ideal loud-speaker characteristics

Loud-speaker (plant) measurements

- Sound pressure.
- position or derivatives: $x_D, \dot{x}_D, \ddot{x}_D$.
- Current i_c (here we consider a voltage driven loud-speaker).

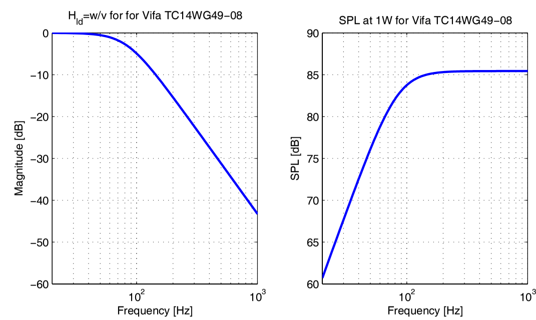
Fixed feed-forward control



- Feed-forward controller = linear filter + inverse plant model.
- $v(t)$: input to controller.
- $u(t)$: input to plant.
- $y(t)$: plant output (plant measurement)
- Assume we know everything including non-linearities.

Controller in detail

- Split in linear and inverse plant model.
- Linear filter: ideal loud-speaker characteristics.



- Master equations can be formulated as a **state space model**.

State space model

$$\dot{\mathbf{z}}(t) = \frac{d\mathbf{z}(t)}{dt} = \mathbf{f}(\mathbf{z}(t)) + \mathbf{g}((\mathbf{z}(t))u(t)$$
$$y(t) = \mathbf{h}(\mathbf{z}(t))$$

- Example – closed box, the master equations

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} i_c \\ x_D \\ \dot{x}_D \end{bmatrix}$$

- $u(t)$: input voltage.
- Output (plant measurement), e.g.

$$y(t) = i_c(t) \qquad \text{or} \qquad y(t) = x_D(t)$$

Controller – inverse plant model

- Transducer converts input voltage $v(t)$ to sound pressure $p(t)$.
- Ideally

$$p(t) \propto v(t)$$
$$p(t) \propto \ddot{x}_D(t)$$

- The **inverse plant model** is derived by taking time derivatives $\dot{y}(t)$, $\ddot{y}(t)$,... of the state space model

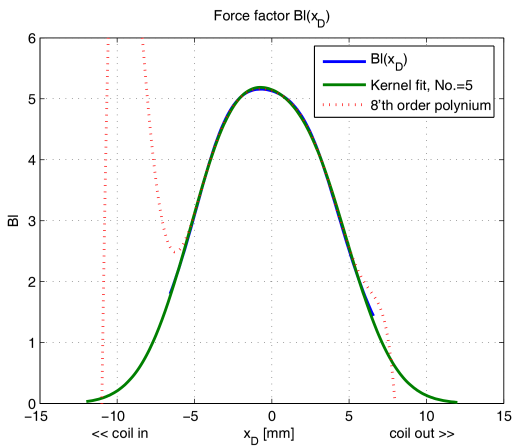
$$y(t) = \mathbf{h}(\mathbf{z}(t)) \quad \text{and} \quad \dot{\mathbf{z}}(t) = \mathbf{f}(\mathbf{z}(t)) + \mathbf{g}((\mathbf{z}(t))u(t)$$

and setting the input voltage

$$\ddot{x}_D(t) = v(t) \quad \Rightarrow \quad u(t) = F^{-1}_{plant}(\mathbf{z}(t), v(t))$$

where $\mathbf{z}(t)$ follows from the state space model.

Fitting the non-linearities I



Fitting the non-linearities – equations

- Polynomial fit

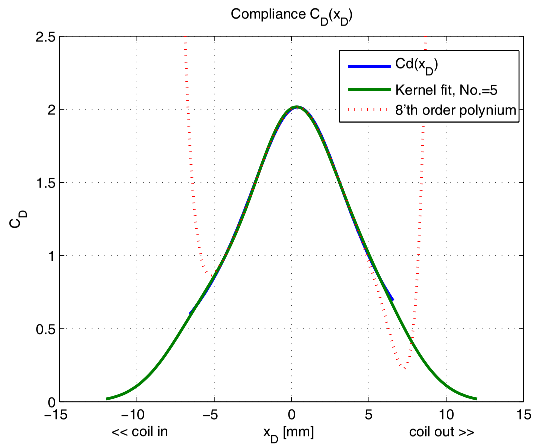
$$\widehat{Bl}(x_D) = \sum_{l}^{N_{basis}} w_l \, x_D^l$$

- Kernel fit

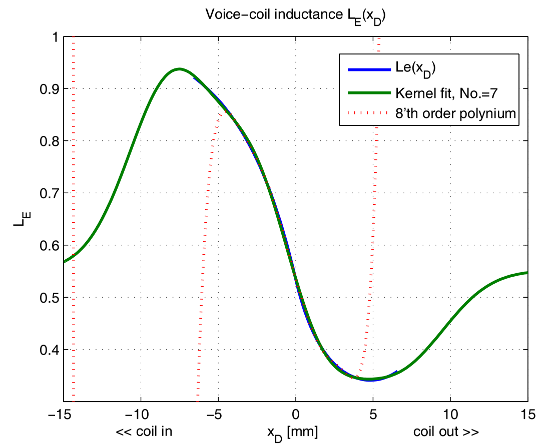
$$\widehat{Bl}(x_D) = \sum_{l}^{N_{basis}} w_l \, \exp\left(-\frac{1}{2\sigma_l^2}(x_D - c_l)^2\right)$$

- To keep simple let parameters appearing non-linearly be fixed: $\sigma_l \sim \frac{range}{N_{basis}}$ and $c_l = [min(x_D), \ldots, max(X_D)]$.
- Simplest solution: least squares.

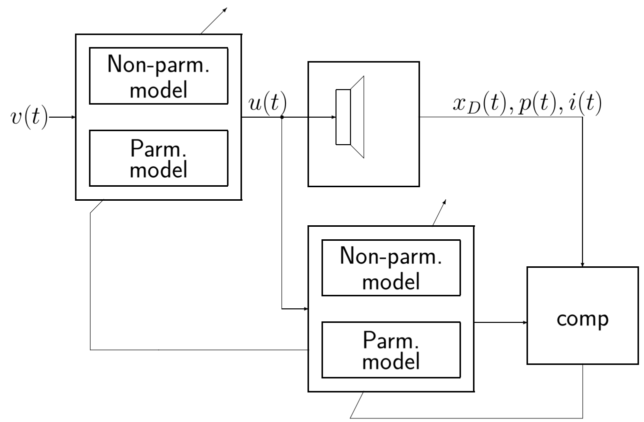
Fitting the non-linearities II



Fitting the non-linearities III



Adaptive feed-forward control



Conclusion

- Modern learning techniques have a potential for
  - better physical models
  - new combined non-parametric/parametric plant models and controllers
  - improved adaptive learning