Overview

- Deliberate nonlinear design: the tradeoffs
- Controller: Architecture & general considerations
- Loudspeaker model: Alternatives & affects on controller
- Constructing a nonlinear compensation controller from a loudspeaker model
- Tuning the controller
- Optimal design with nonlinear control
Typical Microspeaker

- ø15 ~ 20mm x 3~5 mm
- Standard electrodynamic motor
- Usage: Ring tones, FM radio, hands-free telephony
- Key engineering parameters
  - Voltage sensitivity
  - Size (including enclosures)

![Diagram of a microspeaker with dimensions](image)

Deliberate Nonlinear Design: Shortening Coil Height

- Optimisation of coil height
  - Short height provides higher sensitivity
  - Large height reduces nonlinearity, sensitivity
- Simulation of basic parameters vs. coil height
Change in basic parameters vs. coil height

Assumptions:
- Coil length proportional to height
- Constant resonance frequency

Small-signal sensitivity vs. coil height
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General considerations for a controller

- Many audio products require DSP (MP3, dig. cellular telephony)
- Analogue:
  + Can be designed from physical model (s-space; differential eqs.)
  – Drift problems
  – Not (easily) programmable
- Digital:
  + Programmable
  + Cheap (free?) HW – if already required
  – Algorithms very different from physical model
Theoretical foundations for nonlinear compensation

Strategies (theoretical approach) for digital adaptive feedforward controller:

- **Volterra series, discrete-time**
  - Straightforward nonlinear theory: extension of linear system theory
  - Computationally expensive
  - Parameters have no physical interpretation
  - System ID difficult w/o expensive feedback sensor

- **Narmax-model, Neural-Network**
  - Computationally cheaper than Volterra series
  - Parameters have no physical interpretation
  - System ID difficult w/o expensive feedback sensor

- **Feedback linearisation (inverse dynamics model)**
  - Parameters with physical interpretation
  - System ID by indirect (inexpensive) feedback sensor
  - Same computational effort as Narmax model

Architectures for nonlinear compensation

Feedback control

- Force \( y(t) \) to follow \( v(t) \)

Feedforward control

- Model loudspeaker, compensate accordingly

Adaptive feedforward control

- System identification to update feedforward controller
Feedback linearisation

Digital controller design: from plant model
- Continuous time model
  - “Digitised” with numeric integrators
  - Model built from traditional LPM
  - Difficult parameter updating
- Discrete time model
  - Easy digital controller design
  - How to build model? (Ldspk. LPM is in continuous-time)

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Lumped-Parameter Nonlinear Loudspeaker Model

- Combination of electrical and mechanical LPM models
- Acoustic dynamics treated as mechanical-equivalents
- Parameter variation with displacement produces nonlinearity
- Chief culprits:
  - \( \phi(x) \): transduction coefficient
  - \( k(x) \): suspension stiffness
- Nonlinear differential equations may be analysed by numerical integration, other methods.

\[
\begin{align*}
    v_c(t) &= R_{oa}(t) + L_{oa}(t) \frac{dx(t)}{dt} + \phi(x(t)) \frac{dx(t)}{dt} + \frac{d}{dt} \int_0^t R_{oa}(t) dx(t) dt \\
    i_i(t) &= m_0 \left( \frac{d^2x(t)}{dt^2} + c_0 \frac{dx(t)}{dt} + k_0(x(t)) \right) - \frac{1}{2} \frac{d}{dt} \left( \int_0^t R_{oa}(t) dx(t) dt \right)
\end{align*}
\]

System dynamics view

- Linear model

\[
\begin{align*}
    V_c(s) &= \frac{1}{R_{oa}} I_i(s) \\
    I_i(s) &= \frac{1}{m_0 s^2 + c_0 s + k_0} X_A(s) \\
    X_A(s) &= \frac{s^2}{4\pi} P(s)
\end{align*}
\]
Adding nonlinear components

- Added as zero-memory nonlinear systems to linear model

DSP implementation of model

- Linear filters needed for:
  - Mechanical receptance
  - Differentiation
- Other components are “zero-memory”
FIR filter Mechanical Receptance

- FIR inherently stable – attractive for adaptive filtering
- Loudspeaker’s mechanical dynamics are ‘resonant’
  - Inefficiency modelled by all-zero filter

Example electrical admittance impulse response

IIR Filter for Mechanical Receptance

- Resonant behaviour modelled with low-order IIR filter
  - Same structure can be used to approximate differentiation
  - Nonlinear-elements are zero-memory systems: straightforward digital implementation
- Forms basis for feedback linearisation-based design of nonlinear controller
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Inverted digital model

- Algebraic inversion of voltage & displacement described by model
- Basis for nonlinear compensation algorithm
- Describes voltage that would have created some specified displacement

\[
\begin{align*}
    r_{n}[n] & \rightarrow 1/\sigma, \\
    1/\phi(x) & \rightarrow R_{sh}, \\
    R_{sh} & \rightarrow r_{n}[n] \\
    r_{n}[n-1] & \rightarrow k(x), \\
    k(x) & \rightarrow \Sigma, \\
    \Sigma & \rightarrow b_{n-1}, \\
    b_{n-1} & \rightarrow a_{n-1}, \\
    a_{n-1} & \rightarrow \phi(x), \\
    \phi(x) & \rightarrow r_{n}[n].
\end{align*}
\]
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Parametric drift and uncertainty

- Loudspeaker characteristics change with:
  - Manufacturing tolerances
  - Temperature variations

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Symbol</th>
<th>Temperature Variation Coefficient</th>
<th>Manufacturing tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC-resistance</td>
<td>$R_{dc}$</td>
<td>$0.004 \cdot R_{dc} \cdot ^\circ C$</td>
<td>±10%</td>
</tr>
<tr>
<td>Suspension damping</td>
<td>$c_d$</td>
<td>-0.05</td>
<td>±10%</td>
</tr>
<tr>
<td>Suspension stiffness</td>
<td>$k_s$</td>
<td>(none)</td>
<td>±30%</td>
</tr>
<tr>
<td>Transduction coefficient</td>
<td>$\phi_0$</td>
<td>-0.005</td>
<td>n/a</td>
</tr>
</tbody>
</table>

- Parameter drift causes mistuning of feedforward controller
  - Fatal problem for pure feedforward control
  - Controller must be tuned to changes in the loudspeaker

Tracking Changes in the Loudspeaker

- System identification: Tracking changes using adaptive filtering

- Identified parameters are updated in pre-processor
- Measurement of electrical current, mechanical vibration, or acoustic pressure can be measured
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Simulation of effective sensitivity

- Effective sensitivity calculated as a function of coil height calculated

\[ S_{eff} = \frac{1}{C_c} \left. \frac{P_{lim}(s)}{v_c(s)} \right|_{s=0} \]

- \( C_c \) is additional amplifier output required for distortion compensation

\[ C_c = \frac{P_k(u)}{P_k(w)} \]
486Hz

![Graph showing Effective Sensitivity at 486Hz](image)

825Hz

![Graph showing Effective Sensitivity at 825Hz](image)
1074Hz

![Diagram showing effective sensitivity at 1074Hz with various line colors indicating different thicknesses.](image)

3082Hz

![Diagram showing effective sensitivity at 3082Hz with various line colors indicating different thicknesses.](image)
Measurement

- Modified voice-coil height samples prepared for measurement
  - 1.2mm (standard height)
  - 0.8mm
  - 0.4mm
  - 0.2mm

Linear FRF of shortened-height V-C samps.

- Shorter heights provide higher sensitivity
- 0.2mm shows ~10dB higher voltage sensitivity, +4dB power efficiency
Compensation of distortion

Measurements from 0.2mm height coil
- Broken: No control
- Sold: With control

Conclusions
- Deliberate introduction of nonlinearity can increase loudspeaker sensitivity
- Nonlinearities can be compensated by digital processing
- Optimal design point not fully clear